# Why Twelve Notes?

C. Bryzek
cambryzek@gmail.com

May 16, 2026

## I Introduction

A couple of years ago, I read Sun Tzu's "The Art of War", and distinctly remember one line that read "There are not more than five musical notes, yet the combinations of these five give rise to more melodies than can ever be heard."[1] As a modern reader, I recognised this passage as referencing the pentatonic scale, famous for sounding good in almost any combination of notes (provided the musician plays some perceptible rhythm). It started my wondering—as I have been wondering for many years—why we now, in Western art music, have 12 tone scales. Immediately, it should be stated that this is not a new topic: many microtonal scales have emerged and shown up across the world that have explored fitting more notes into the octave than 12.[2] Here, I briefly discuss some thoughts I have had and come across on the topic and a couple of related ideas over the last several years.

As I originally learned, the reason we have 12 notes in the scale is because, after stacking perfect fifths 12 (unique) times, we end up back at our tonic. Say we start on the note A. Our progression would then look like:

1A – 2E – 3B – 4F♯ – 5C♯ – 6G♯ – 7D♯ – 8A♯ – 9F – 10C – 11G – 12D – A.

This wasn't perfectly satisfactory to me. If done properly, the final pitch that we arrive at should be a perfect power of two multiple of the fundamental frequency. Performing this process 12 times, though, we end up with a frequency multiple of

$$(3/2)^{12} = 129.746$$

times the frequency of the tonic (the note representing the fundamental frequency). This multiplication factor 129.746 is *close* to $128 = 2^7$, but it is not exact.[3] Based on the success of Western music traditions, it is clear that this is close enough produce wonderful music, but stacking fifths and saying we're *close* did not feel like a perfectly satisfactory answer to why *12*. Could we have had 11 notes by accident? 13? It turns out that 7 notes approximates $2^4$. Why didn't we stop there?

---

[1] Sun Tzu, *The Art of War*. Trans. Lionel Giles (Allandale Online Publishing, 2000). Originally written c. 5$^{th}$ century BC.

[2] *Wikipedia*. "Microtonality."

[3] In fact, other than the trivial case of 0 notes in the scale, we can never get back *exactly* to our tonic by stacking fifths.

Through the rest of this article, I bring together a couple topics of interest to provide a more convincing argument for why 12 notes to a scale may be special. Section II sets the foundation for this discussion by describing the physical relevance of the octave and the fifth, and a preliminary reason for 12 notes. Section III discusses mathematical approximations of the fifth, where a more compellingly aesthetic reason for 12 notes presents itself. In light of this mathematical background, Sec. IV briefly describes the natural existence of alternate microtonal scales.

# II The Octave and Fifth

When we discuss musical scales, we are describing a repeated spacing of notes that can be moved around the range of human hearing to create a recognisable musical pattern. One natural place to define the repetition of a scale is when the fundamental frequency is doubled. For instance, if the fundamental pitch for a certain scale is 440 Hz, then the octave will have a frequency of 880 Hz. For those accustomed to Western art music, we perceive this doubling of frequency as the same pitch, just at a higher frequency, and ascribe letter names to both the fundamental and the octave (A4 and A5, in this case).[4] Equal temperament scales divide up each octave into a number of equally logarithmically spaced notes, with the familiar 12-tone equal temperament dividing the octave up into 12 notes spaced by $\sqrt[12]{2}$ multiples (so, a half step above 440 Hz would be $\sqrt[12]{2} \cdot 440$ Hz = 466.16 Hz, which is an A♯4). Before discussing scale splitting, however, this section addresses a couple implications of the octave having exactly double the fundamental's frequency, beginning with the overtone series then visualising the overlapping of periods of the fundamental and the octave, and applying that alignment to other intervals to think on the impact of "beats" on consonance and dissonance.

## i Overtone Series

The overtone series describes the additional pitches heard above any played fundamental pitch. When one bows an A4 on a violin, for instance, not only do we hear 440 Hz, but also many more overtones that ring (albeit with much lower amplitudes) above that fundamental.[5] In terms of scale degrees, the overtone series is

$$1 - 1 - 5 - 1 - 3 - 5 - \flat 7 - 1 - 2 - 3 - \sharp 4 - 5,$$

where 1 is standing in for both the fundamental and octave. Most relevant to this discussion are the first three pitches in the overtone series, which give us the fundamental, octave, and perfect fifth (which shows up as three times the fundamental frequency in the overtone series). Whenever we hear *any* note, we also hear all of its overtones, and the most prevalent frequencies are that octave and fifth, suggesting some importance in connection to the fundamental.

[4] This perception of "octave equivalence" may not be universal cross-culturally. See Jacoby N, Undurraga E, McPherson M, et al., "Universal and Non-universal Features of Musical Pitch Perception Revealed by Singing." *Current Biology* 29, 3229–3243.e12 (2019).

[5] *Integrated Music Theory*. "Lesson 8a: The Overtone Series."

## ii Period Alignment

This importance of the octave and fifth can be visualised by overlaying the waveforms of these intervals with the fundamental, as shown in Fig. 1. There is no *new* information gleaned here—we already knew that these pitches have frequencies that are nice integer multiples of the fundamental, so of course they line up—this is merely a way of seeing the same information graphically. The octave is plotted in green, the fifth in blue, and vertical lines display each time the period of one of these intervals lines up with the fundamental. Note the regularity of periodic alignment for these two intervals: the octave lines up with each period of the fundamental, the fifth with every second period.

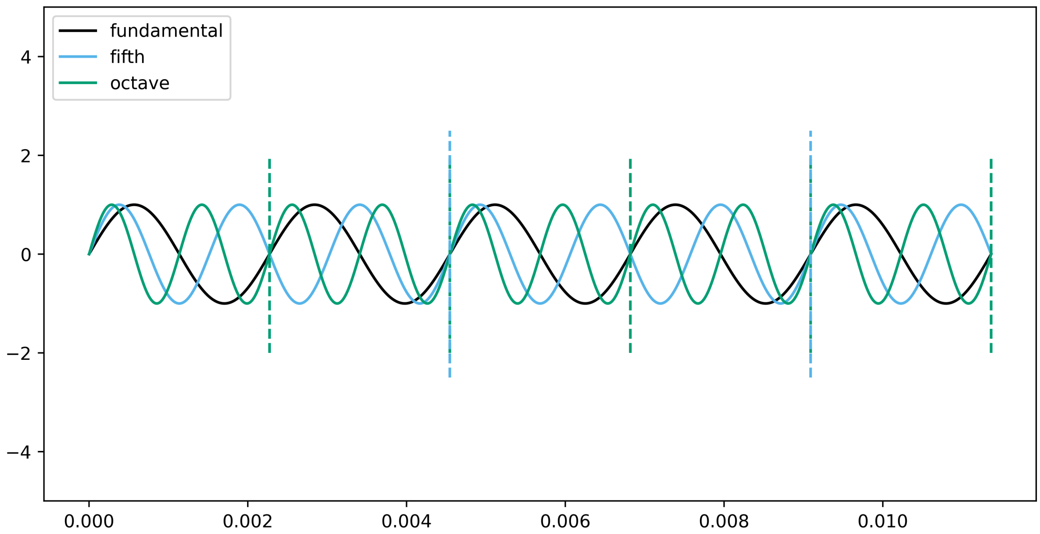


Figure 1: Periodic alignment of the octave and fifth with the fundamental. Vertical dashed lines display each time the period of a given interval aligns with the fundamental's period.

If we instead plot notes considered to be dissonant in Western music culture—such as the minor second and the tritone against the fundamental, as shown in Fig. 2—we find much less frequent alignment. The minor second lines up with every 15$^{\text{th}}$ period of the fundamental, and the tritone every 32$^{\text{nd}}$!

There is a *long* conversation to be had here on the implications of this period alignment on consonance and dissonance,[6] but the most interesting result to me is a short discussion of beats. When two frequencies are played together, we hear not only the two tones, but also the beat frequency created by their combination. This is a third frequency that describes how often periods line up, and, in general, the longer the beat frequency, the more dissonant an interval sounds. For extremely close frequencies (say, 440 Hz and 442 Hz), this beat creates a generally undesirable warbling sound.[7] Intervals such as the octave and fifth with relatively short beat frequencies are generally perceived as pleasant and consonant. An example of this beat frequency is shown in Fig. 3, where the top displays the overlay of fundamental and fifth, and the bottom shows the combined wave created by their interference, along with

[6] For a start on this discussion, see MinutePhysics. "The Physics Of Dissonance." *YouTube*, 2025.
[7] To hear this warbling, see Tony Verheyden. "Beats at 2 Tuning Forks." *YouTube*, 2020.

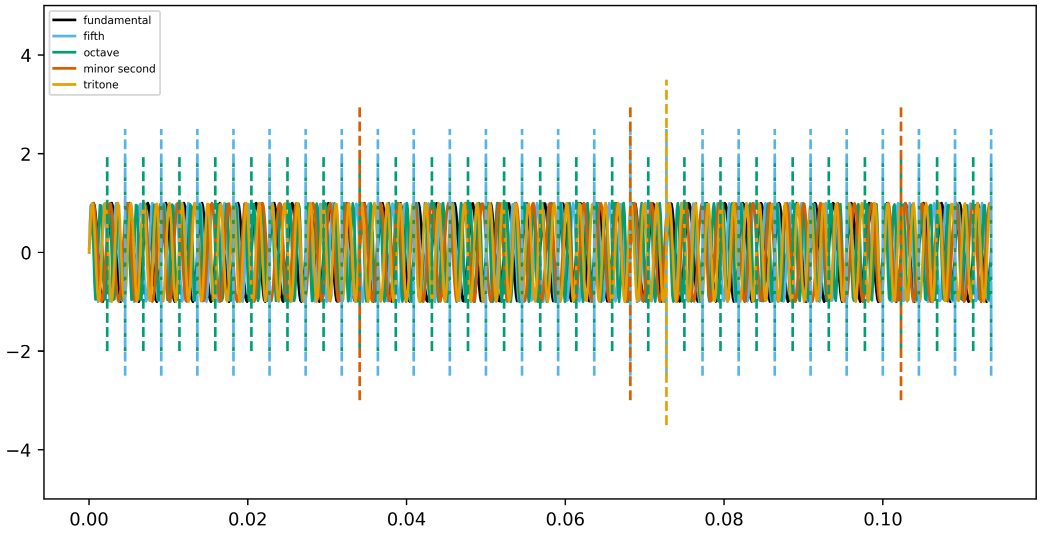


Figure 2: Periodic alignment of the octave, fifth, minor second, and tritone with the fundamental. Vertical dashed lines display each time the period of a given interval aligns with the fundamental's period.

the beat frequency envelope. Note that the anti-nodes of the combined wave in the bottom frame align perfectly with the period alignment bars in the top frame.

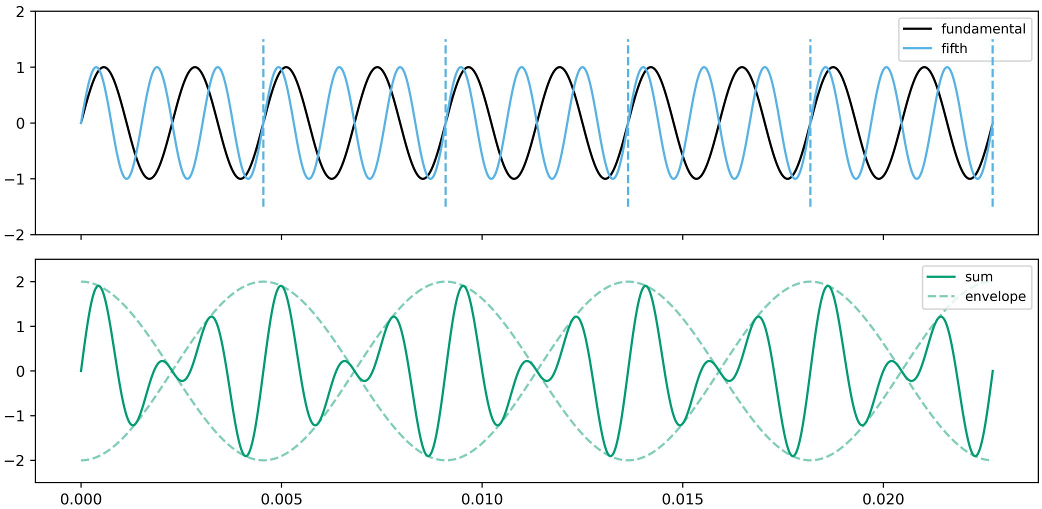


Figure 3: Beat frequency created by combination of the fundamental and fifth. The top panel shows the two separate wave forms, and the bottom shows their combination from interference with the beat frequency envelope.

## III Approximating the Fifth

It is evident that both the octave and the fifth, by virtue of their frequency ratios, have a powerful relation to the fundamental frequency. In designing a universal scale, then, it may be important to use the octave as a framework for where the scale should repeat itself, and include the fifth as an important note that we wish to include.[8] For the construction of an equal temperament scale of $n$ notes, we are therefore looking for what number of notes allows us to step from tonic of frequency $f_0$ to octave ($2f_0$) with a good approximation of the fifth, $3f_0/2$. Mathematically to get the octave, our frequencies will be spaced by ratios of $\sqrt[n]{2}$, and after $n$ steps we will arrive at $2f_0$. We want to find, then, what values of $n$ lead to at least one note in the scale having a radio to the fundamental frequency of 3/2. The values for $n$ that give such approximations are summarised in Tab. 1, which displays the $n$ value for each, the ratio of the frequency of the fifth approximation to the fundamental, and the cents deviation from a perfect ratio of 3/2, where 1 cent represents 1/100 of the logarithmic space between consecutive chromatic scale degrees in a given scale.[9]

Note that these values for $n$ represent *evenly* spaced note values—these are equal temperament systems. So, even though $n = 7$ works out nicely, in this case this does not represent the familiar ionian scale, but would likely sound microtonal to our ears. From observing the rest of Tab. 1, we can see that there are many possible numbers of notes that we can fit into a scale with a good approximation of the fifth. For instance, why 12 notes, instead of 17? Here, I have found an answer that is satisfactory enough for myself: we want a scale that affords us enough range of possibility for musical expression without being impossible to navigate (humans usually only have ten fingers after all). This roughly limits us to either $n = 7, 12,$ or 17, within which 12 approximates the fifth the best. This, for me, at least answers why 12, rather than 11 or 13. Looking at how we get a perfect fifth removes ambiguity within a range of ±5 of our familiar 12-tone equal temperament. Last, I find it aesthetic that, of the first four possible scale numbers listed here, 12 is the only composite value.

[8] Of course, not all scales will use the fifth, as artistic freedom demands setting oneself free of these conventions. For a general scale that intended for full musical creativity, however, it is nice to have the *option* of the fifth, rather than have no fifth interval at all.

[9] For a continuation of this sequence beyond $n = 100$, see *OEIS*. "A060528." *The On-Line Encyclopedia of Integer Sequences*.

Table 1: Equal temperaments with $n$ chromatic degrees containing a good approximation of a perfect fifth (ratio 3/2) for $n \in [1, 100]$. Columns display the ratio of the closest fifth to the fundamental, along with the cents deviation from a perfect fifth within the specified scale. Scales with a fifth within five cents of perfect are bolded.

| $n$ | Ratio to Fundamental | Cents Deviation |
|---|---|---|
| 7 | 1.48599 | -9.5 |
| **12** | **1.49831** | **-2.0** |
| 17 | 1.50341 | 5.6 |
| 19 | 1.49376 | -11.4 |
| **24** | **1.49831** | **-3.9** |
| **29** | **1.50129** | **3.6** |
| 31 | 1.49552 | -13.4 |
| 34 | 1.50341 | 11.1 |
| 36 | 1.49831 | -5.9 |
| **41** | **1.50042** | **1.7** |
| 46 | 1.50207 | 9.2 |
| 48 | 1.49831 | -7.8 |
| **53** | **1.49994** | **-0.3** |
| 58 | 1.50129 | 7.2 |
| 60 | 1.49831 | -9.8 |
| 63 | 1.50243 | 14.7 |
| **65** | **1.49964** | **-2.3** |
| 70 | 1.50078 | 5.3 |
| 72 | 1.49831 | -11.7 |
| 75 | 1.50177 | 12.8 |
| **77** | **1.49943** | **-4.2** |
| **82** | **1.50042** | **3.3** |
| 84 | 1.49831 | -13.7 |
| 87 | 1.50129 | 10.8 |
| 89 | 1.49928 | -6.2 |
| **94** | **1.50015** | **1.4** |
| 99 | 1.50093 | 8.9 |

## IV Possibilities for Other Scales

While I am no expert on microtonal scales, it is striking to me how many values of $n$ listed in Tab. 1 show up as common microtonal scales. The EDO system provides one way of systematically dividing up the octave into a set of $n$ notes, and some of the more commonly used scales are

$$\mathbf{19}, 22, 23, \mathbf{24}, 26, 27, \mathbf{29}, \mathbf{31}, \mathbf{34}, \mathbf{41}, \mathbf{46}, \mathbf{53}, \mathbf{58}, \mathbf{72}, \text{ and } 96 \text{ EDO},$$

bolded if they appear in Tab. 1.[10]

Clearly, there is *some* meaning to the 12 tone equal temperament system, even if it was not intentional in the original formulation of Western harmony. While there is nothing new presented here, I have collected a variety of thoughts I have had on the subject of scales, waves and harmony, and microtones. I have recently been increasingly pleased by microtonal harmony, and encourage everyone to explore these alternate scales and move beyond any initial discomfort we may have from encountering something new and different. Below are some music examples I love that utilise microtonal scales or harmonies.

- Maddie Ashman and Tolgahan Çoğulu, "King Gizzard's Billabong Valley" (YouTube, 2022)
- Buzz Gravelle, "The River Has Many Voices" (YouTube, 2022)
- Stephen Weigel ft. Clarissa, "Fly Me to the Moon (29-TET)" (YouTube, 2026)
- KEXP, "Angine de Poitrine — Full Performance" (YouTube, 2026)
- Bryan Deister, "improvisations in 1edo through 50edo" (YouTube, 2025)

## Acknowledgments

Thank you to Leo Grossman for answering several questions I've carried around on this topic for a couple of years, and for motivating Sec. III.

[10] *Wikipedia*. "Equal temperament: Other equal temperaments.".